

BOOK REVIEW†

Stability and Transition in Shear Flows. By P. J. SCHMID & D. S. HENNINGSON.
Springer, 2001. 556 pp. ISBN 0-387-98985-4. £59.50 or \$79.95
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While our knowledge of linear stability theory is by now fairly complete, no full understanding of the turbulent breakdown mechanism has yet been achieved. However, the past two decades have seen a number of successful attempts to bring instability theory a little closer to explaining the onset of turbulent behaviour. The book by Schmid & Henningson does precisely this: starting from classical stability theory, the authors present mostly recent developments, consider situations with higher complexity and eventually leave us at the brink of turbulence – as close as one can get at the time of writing.

Some familiarity with basic fluid dynamics and wave-like phenomena is assumed and the material is thus primarily intended for graduate students and beyond. As a fairly up to date account of issues between stability and turbulence, it will help those coming from any branch of fluid dynamics to quickly gain an overview of the latest techniques and results. Throughout, a variety of mathematical and numerical tools is introduced, each illustrated in different situations to show a range of possible applications. While some chapters serve as reference material, e.g. the final chapter on transition scenarios, it is also an excellent read from cover to cover!

The book starts with a brief introduction of governing equations and basic stability/instability, linear/nonlinear temporal/spatial concepts. Whereas the first part of the book (Chapters 2–5, about one third of the total) is focused on fundamental concepts of hydrodynamic (mostly temporal) stability theory, richly exemplified using simple parallel shear flows, the second part (Chapters 6–9) builds on these results and is devoted to more realistic, and hence more complex, situations such as three-dimensional boundary layers, spatially evolving flows, secondary instabilities and transition.

Chapter 2 is concerned with linear inviscid analysis, and illustrates the fundamental tools and results that are prerequisites for any serious shear flow analysis: modal solutions, Rayleigh's and Fjørtoft's criteria, the method of stationary phase, Laplace's transform. While linear dispersion relations and the associated eigenfunctions can be found in any standard textbook, particularly illuminating here is the explicit treatment and the corresponding illustrations of the evolution of a point-like disturbance in a boundary layer.

In Chapter 3 the stability analysis of two-dimensional parallel shear flows is carried a step further by keeping viscous terms, solving the resulting Orr–Sommerfeld and Squire equations and explicitly obtaining the spectra and eigenfunctions for Couette, pipe and Blasius boundary layer flows. In the limit of large Reynolds numbers, asymptotic techniques and critical layers are also briefly mentioned. Particularly helpful is the discussion of the sensitivity of numerically computed eigenvalues and its relationship to pseudospectra.

† This is a corrected version of the review that first appeared in volume 483, pages 343–345.

Chapter 4 further analyses linear viscous temporal instability with emphasis on the initial value problem rather than on the eigenvalue problem. In the first part of the book, this is certainly the authors' favourite chapter: topics such as non-normal operators, transient growth, optimal growth, and optimal disturbances are covered in great detail, illustrated by numerous examples drawn from the authors' own work.

The first part concludes with Chapter 5 on nonlinear stability of which, obviously, no general theory exists. Besides classical techniques such as weakly nonlinear expansions, wave interactions and bifurcation analysis, the chapter includes an enlightening discussion on nonlinear equilibrium solutions and provides a numerical strategy to compute them.

The second part of the book starts with Chapter 6 on the Falkner–Skan(–Cooke) velocity profiles: the drosophila of boundary layers with pressure gradient and crossflow. Then additional features due to body forces (related to system rotation or streamline curvature), surface tension or compressibility are analysed with the tools of the first part. In contrast, the section on unsteady flows introduces particular techniques to investigate stability of basic flows that are periodic in time or even display arbitrary time dependence.

After the so far very detailed coverage of many aspects of temporal stability, Chapter 7 now addresses growth of disturbances in space. This chapter, totalling 120 pages, is certainly the core of the book – besides being my favourite. By resorting to several model problems before addressing the full Navier–Stokes equations, the authors emphasize the particular difficulties associated with the spatial setting. Indeed, it is too often believed that temporal results can always be converted to spatial ones by a simple transformation. One may hope that the two pages on Gaster's transformation, clearly delimiting the validity of this method, will put an end to its erroneous use. Then, the evolution of perturbations in both space and time is addressed by introducing the concept of convective and absolute instabilities: features which are of primary importance to any non-Galilean-invariant open shear flow. After a review of the classical Briggs' method and the more practical cusp map procedure, these concepts are illustrated by considering the two-dimensional wake behind a cylinder and the three-dimensional boundary layer due to a rotating disk: two situations where much understanding has been gained by absolute instability analyses. The chapter continues with a treatment of the spatial initial value problem and the associated discussion of upstream and downstream responses to localized harmonic forcing. The authors then incorporate non-parallel effects, which are nearly always present in realistic shear flows, first by asymptotic multiple-scales analyses and then by parabolized stability equations, exemplified by Görtler vortices and the Blasius boundary layer (the extremely brief presentation of the obscure triple-deck theory is however not very illuminating). Then follows the spatial analogue of optimal disturbances and a brief review of global instability results, as derived from local stability analysis within the assumption of weakly diverging flows. The chapter closes with the receptivity problem, putting into practice some of the tools introduced above.

Chapter 8 brings us a step closer to transition by studying secondary instabilities that may, in turn, affect a finite-amplitude state resulting from a primary instability. The mathematical tool of choice is here Floquet theory, now usable in realistic situations due to the recently available computing power. The various types of secondary instabilities are discussed for Tollmien–Schlichting waves, streaks, Görtler and crossflow vortices, and the special case of Eckhaus instability.

The concluding Chapter 9 is based on the results and techniques gathered in the previous chapters and considers how the various instability mechanisms can trigger

transition to a turbulent regime. In contrast with the rest of the book, very few equations are introduced: the emphasis here is not on mathematical analysis but on describing the many possible routes that the complicated transition process may follow. This final chapter appears to be a quite comprehensive review (besides being an excellent read) of transition scenarios prevailing for two-dimensional waves, streaks, separation bubbles, Dean, Görtler and crossflow vortices etc. The chapter closes with a brief account of different models that have been used, more or less successfully, to predict transition.

On the technical side, one could criticise many overlarge or extremely small figure labels, several very badly placed equations (e.g. p. 238) and some inconsistencies in the layout. But rather than the authors, the publisher is to blame: its only contribution to this work being a photocomposed copy prepared from the authors' \TeX files.

All in all, within its field, this is an extremely complete and well documented book – one which will be wanted in the library by all and on the desk (not on the shelf) by many.

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